Keynes's *Treatise on Probability* at 100 years: its most enduring message

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Abstract

On the occasion of the assessment of the enduring influence of Keynes's *Treatise on Probability* at 100 years, the paper focuses on its relevance for decision theory. The paper places emphasis on Keynes's introduction of the epistemic notion of probabilities that often are non-numerical, as a theoretical object intended to replace frequency probabilities. The paper argues that, as non-numerical probabilities make it possible to deal with uncertainty as if individuals were endowed with interval-valued probabilities, Keynes's 1921 critique of contemporary frequency probability theory turns out to be relevant also with regard to the yet to be established subjective probability theory. Although non-numerical probabilities were used by Keynes to criticize the contemporary application of probability to conduct, it must be acknowledged that, still today, they may constitute an appropriate tool for decision-making when confronting uncertainty, as he hinted at in his late 1930s correspondence with Hugh Townshend.

1. Introduction

In 1937, the year after the publication of the *General Theory* (henceforth GT), Keynes defended his theory from criticism in the *Quarterly Journal of Economics*. Instead of getting involved in a detailed examination of his American critics' comments, he provided a summary of the "simple fundamental ideas" that underlie the GT. As is well known, Keynes was eager to emphasize the revolutionary nature of his theory. But when he summed up what was revolutionary about it, he used the 1937 essay to argue that this amounted to his theory being relevant for a world in which "we have, as a rule, only the vaguest idea of any but the most direct consequences of our acts". Keynes (1937, 213-214) made his point straight: "I accuse the classical theory of being itself one of those pretty polite techniques which tries to deal with the present by abstracting from the fact that we know very little about the future".

Notwithstanding Keynes's insistence on an uncertainty that was to be classified as true, radical, sheer and so on, both the GT and the following essay offered only verbal arguments about the implication of such an uncertainty. Indeed, some commentators have argued that the ensuing

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attempts to model Keynes's theory missed the point of the GT exactly because Keynes's argument is that no precise model can be specified. The so-called fundamentalist interpretation of Keynes rested on this ambiguity between Keynes's challenge of the foundations of "classic" price theory and his methodological rejection of precise mathematical reasoning as a tool for economic analysis. As a result, Keynes's revolution was mainly seen as overthrowing the type of rational choice theory on which the classical thought was based without actually proposing a formalized alternative to it (Backhouse 2006, 25-27).

However, the issue of how to deal with uncertainty also emerged in his correspondence with Hugh Townshend in the same years just after the GT. As argued elsewhere (Zappia 2015), the exchange provides a methodological viewpoint on decision-making that is peculiar for its interest in the analytical content of the issue at stake. The correspondence between Keynes and Townshend, in fact, concentrates on technical issues, providing evidence for the fact that Keynes still had a keen interest in a problem left unsolved in his 1921 *Treatise on Probability* (henceforth TP), namely, the definition of a criterion for decision-making. Such a criterion was needed because Keynes found it unsuited for uncertain contexts what he termed "normal ethical theory" in the TP. Later, in the GT and after, he made similar, and much more noted comments on "strict mathematical expectation" and the "Benthamite calculation of a series of prospective advantages and disadvantages, each multiplied by its appropriate probability" (Keynes 1937, 214). With the benefit of hindsight, one can argue that Keynes was searching for an alternative to what we now call expected utility theory, well ahead of its precise definition.

In what follows the technical thread going from the TP to the GT and after is reviewed in reverse chronological order. After a brief reminder of how mainstream decision theory developed, it is argued that the most enduring message of the TP is to be found in the formal background it provides for a theory of decision-making under uncertainty which objects to the strict subjectivist theory — later classified as Bayesian decision theory — of Frank Ramsey, Bruno de Finetti and Leonard Savage. Well before its consolidation as the mainstream view of modern decision theory, a crucial aspect of the subjectivist viewpoint was implicitly rejected by Keynes. As a result, Keynes's TP can be considered as the book giving birth to a tradition of thought that, though it remained a minority viewpoint for long, informs the intuition of today's criticism of Bayesianism as developed in the so-called ambiguity literature (Machina and Siniscalchi 2014).

2. Mainstream decision theory

In his foundational work of rational choice theory and mainstream decision-making, while defining the axiomatic bases on which uncertainty is reduced to risk, Savage (1954, 21) observed:

> There is some temptation to explore the possibilities of analysing preference among acts as a partial ordering, that is, … admitting that pairs of acts are incomparable [thus rejecting the completeness axiom P1]. This would seem to give the expression to introspective sensations of indecision or vacillation … My own conjecture is that it would prove a blind alley losing much in power and advancing little, if at all, in realism".

And while still commenting on the significance of his axioms, Savage (1954, 39) added

> It would however be disingenuous not to mention that some who have worked on a closely related concept of probability, notably Keynes and Koopman, would object to axiom P6' precisely because it implies that the agreement between numerical probability and qualitative probability is strict.[1]

The possibility to derive a representation theorem proving even under uncertainty what von Neumann and Morgenstern had proved for risk, namely, the equivalence between the maximization of an expected utility function and the acceptance of some basic axioms, motivated Savage's normative approach. His perplexities towards a more realistic slant was endorsed by mainstream decision theory and Keynes's objections that not all probabilities agree with a numerical representation remained part of a minority viewpoint (Levi 1974)

De Finetti, who shared with Savage the aim to provide subjectivist bases for decision-making under uncertainty, had already made a similar comment on Keynes. In his comments on the Cambridge school of probability theorists, who he praised for its epistemic turn away from classic frequentism, de Finetti (1985 [1938]) noted that Keynes's insistence on non-numerical probability estimates could not be ignored. He identified cases in which it is sensible to admit qualitative probabilities exactly because it may be difficult to assign precise numbers. But he nonetheless moved on to an approach in which the application of the so-called Dutch Book argument for consistency in probabilistic estimates would entail sharp probabilities in every instances (Feduzi, Runde and Zappia 2014).

Even earlier, it was Ramsey, the only father of subjectivism Keynes had to confront personally, who explicitly objected to Keynes's approach. Keynes had maintained that the logic of partial implication from evidence to a proposition about events has a similar standing to the logic of

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[1] Savage's P1 implies completeness in the domain of acts, decisions whose consequences are contingent on the realization of a state of nature, as is standard in choice theory. Savage's P6' is usually termed a "technical" axiom, but it implies connectedness of the state space, thus making it possible to assume that the space can be partitioned into an arbitrarily large number of equivalent subsets.

implication of mathematics, so that probability is concerned with "the degree of belief which is *rational* to entertain in given conditions" (Keynes 1973 [1921], 4).[2] Ramsey (1978 [1926]) argued that we would not subscribe to such a representation of degrees of belief, since it would imply not only subjectively consistent but also objectively determined probabilities. In a similar way to de Finetti, Ramsey also used the betting argument to argue that probabilities must be precise numbers, disparaging Keynes's reference to non-numerical probabilities.

It must be reminded, therefore, that the modern approach to subjective probabilities and decision-making under uncertainty developed as a way to move forward in the application of the epistemic viewpoint revived by Keynes, although in a different direction with respect to the logical interpretation Keynes, and later Rudolf Carnap, intended as significant (Hacking 1979, 73). Moreover, Keynes's insistence on the elusiveness of beliefs and the related necessity to allow for a qualitative probability representation of beliefs was took into account, but rejected as irrelevant in terms of gains in realism.

3. Ellsberg's criticism and the Treatise on probability

Notwithstanding early criticism by Maurice Allais (1953), the mainstream approval of subjective expected utility was immediate after Savage's axiomatic foundations. The application of Bayesian analysis to both game theory and management problems soon became pervasive (Luce and Raiffa 1957, Marschak 1964). Savage's perspective mostly remains the mainstream view in rational choice theory and economics in general even today (Blume and Easley 2008).

But despite general approval, Savage's construct of ideal rational agents maximising their subjective expected utility under both risk and uncertainty was also subjected to criticism. A few scholars who accepted the subjective approach argued nonetheless that the behavioural bases of Savage's theory were unconvincing. Specifically, a notable counterexample to the theory was proposed by Daniel Ellsberg (1961) who devised a simple decision context in which decision-makers were shown to violate the axioms of subjective expected utility.

The so-called Ellsberg Paradox suggests that rational agents of the kind proposed by subjectivist do take into account the quality of information at hand when making decisions, namely, they consider whether they feel comfortable in their assessment of probabilities or not. Ellsberg proved this to be in contrast to what Savage had assumed and it had since been taken for granted in

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[2] This "logical" necessity of probability characterized Keynes's (1973 [1921], 19) approach: "our rational belief is … subjective in the sense of being relative to the individual. But given the body of premises … the conclusions, which it is rational for us to draw, stand to those premises in an objective and wholly logical relation".

rational choice theory. For Ellsberg, his paradox had to be considered as a conclusive counterexample to the argument that all uncertainties ought to be subsumed to risks. Today's critics of the Bayesian approach have taken stock of Ellsberg's argument to propose an array of non-Bayesian decision theories (Gilboa 2009)

In his article, Ellsberg referred to Knight's *Risk, Uncertainty and Profit* arguing that there are probability relations about which decision-makers feel less 'sure' as compared to others, and that this fact shows that not all uncertainties can be reduced to risks. But later, while providing the methodological bases of his theory in his 1962 doctoral thesis, Ellsberg (2001 [1962]) mostly referred to Keynes's *Treatise on Probability* which he identified as providing an even more relevant and comprehensive analysis. Keynes's understanding of probability in the TP, he claimed, provides the right framework for a critique of the mainstream approach to decision-making under uncertainty.

Ellsberg's criticism of the subjectivist mainstream is normally related to his introduction of "ambiguity", the notion he used to identify situations that could not be simply addressed as if they were risky ones. But his main aim was to investigate situations of uncertainty in general (Zappia 2021). Moreover, Ellsberg wanted to investigate how a possible alternative formal representation of degrees of beliefs in uncertain situations could be devised. And he recognized that Keynes's probabilistic approach was particularly significant to a constructive analysis of decision-making since Keynes's *TP* had considered "formally the notion of *non-comparability* of beliefs," as a consequence of the rejection of the frequentist assumption that there always exists numerically precise probabilities (Ellsberg 2001 [1962], 9-11).

There is of course an obvious analogy between Ellsberg's ambiguity and Keynes's weight of argument: for Ellsberg, as for Keynes, the vagueness of the decision environment shapes the probability function representing degrees of beliefs, the lower the weight the higher the possibility that the usual properties of probabilities are not satisfied. But Ellsberg insisted that Keynes's epistemic approach to probability was crucial because it originated a tradition of thought that openly questions the Bayesian assumption that standard probabilities adequately represent situations where information is perceived to be vague and, therefore, it does not take for granted that probabilities are always comparable. Ellsberg argued that Keynes's discussion of qualitative, non-numerical probabilities inspired Bernard Koopman (1940) and Irving Good (1962) in their investigations of an axiomatic structure compatible with interval-valued probabilities generating a partial, incomplete ordering. Unlike Ramsey, de Finetti and Savage, Ellsberg maintained, the theory of probability put forward by Keynes and his followers constitutes "a theoretical approach that admits vagueness as an explicit factor without apology and provides a formal vocabulary for discussing it" (Ellsberg, 2001 [1962], p. 10).

4. Keynes on decision-making under uncertainty

As noted in the introduction, in his GT and later Keynes highlighted uncertainty as the main issue he intended to raise against "Classical Economics". A crucial interpretative issue, therefore, is what Keynes had actually in mind for the behaviour of economic agents in situations of uncertainty. In particular, it is unclear what kind of decision-making techniques he may have had in mind when indicating that, although decision theory has to address "the fact that we know very little about the future", he was nonetheless interested in a theory that "should not itself submit to market-place idols". After concentrating on the way individuals are forced to behave in markets where fundamental values may be of limited use, due to the strength of short-term speculation, Keynes (1937, 223) reminded the reader:

> Though this is how we behave in the market place, the theory we devise in the study of how we behave in the market place should not itself submit to market-place idols. I accuse the classical economic theory of being itself one of these pretty, polite techniques which tries to deal with the present by abstracting from the fact that we know very little about the future.

Certainly, such a claim suggests that there must be at least a few Keynesian agents who reject the attitude of falling back on conventions and relying on imitative behaviour (Zappia 2016).

Keynes's uncertainty in the GT is mainly seen as a question of how to act in markets driven more by the conventional attitude to adhere to actual market values than by fundamental values. For many scholars working in the Keynesian tradition, Chapter 12 of the GT shows that the rationalistic approach of the TP is mostly put aside in favour of an analysis of the determinants of conventions, as when the impossibility to formulate a quantitative probability makes it impossible to calculate the future returns of an asset and expectations limit the inclination of entrepreneurs to invest in new capital (Dow 2003). However, there remains the issue of how alert individual agents should behave in markets whose outcomes are influenced by harmful conventions, and that are continuously on the brink of failure (Gerrard 2003). What kind of criterion for decision-making would they adopt? And, even more relevantly, what should be the rule followed by policy makers while making decisions? It would prove difficult to argue that Keynes would have envisaged for "enterprise" what he had for "speculation", namely, the choice to adapt to conventional, but in principle unjustified, market outcomes, even in the face of sheer uncertainty (Basili and Zappia 2021).

While it is true that there is almost no reference in the GT to probability, and that there is likewise almost no reference to uncertainty in the TP, the link between the two notions is apparent. In the GT the state of long-term expectation taken into account by investors is unduly influenced by the short-term attitude of speculators in the stock market because it is difficult, if not impossible, to

give a probabilistic representation of beliefs that concern a distant future. The TP providing an in depth analysis of what probability theory can encompass, it seemed obvious to many commentators that, after many years of lack of interest in it, the TP had to be investigated to determine what the actual meaning of uncertainty in the GT was (Lawson 1985).

5. Keynes on probability

In his TP Keynes questioned what contemporary frequency probability theory could embrace. Knowledge of the relative frequencies of an outcome in a series of otherwise similar outcomes is sometimes available, but the assumptions for such empirical knowledge to be valid are seldom met, while a general theory of probability should be able to represent degrees of belief in a much larger class of situations. Henceforth, Keynes put forward an alternative, epistemic notion of probability. He aimed at a logic of "rational" degrees of belief, that he presented as a theory of the objective value subjective probabilities should entail: every rational individual would "perceive" − as Keynes put it and Ramsey famously criticized − the same degree of probability of the relation between the same evidence and a proposition. This view was superseded by the purely subjective approach of "personal", as Savage used to call them, degrees of belief.

Even in view of his brief comments on Ramsey, to whom Keynes paid tribute in a chapter of his *Essays in Biography*, it appears difficult to claim that Keynes stayed loyal to this logical, necessary view of probability. But seen in retrospect Keynes's analysis in the TP has much wider significance than that of a specific, possibly out-dated, philosophy of probability. His logic of comparative probability neither depends on the original philosophical background in which Keynes presented it, nor it can be subsumed to a strict subjectivist perspective (Runde 1994). Indeed, as argued in Basili and Zappia (2009), Keynes's critical remarks on frequency probability in the TP constitute a challenge to any theory of probability which is based on a unique additive distribution, even the current subjective Bayesian mainstream.

To be sure, as recently restated by Carabelli (2021), there is a strong critical part of Keynes's argument against the application of standard probability theory in the economic theory of uncertainty. However, there is also a positive, constructive component of the TP that is at least as relevant, but has received comparatively less attention. To this constructive component, it is the main argument of what follows, Keynes did remain loyal.

The structure of the TP makes it clear that Keynes's view of the potential incomparability of alternative probability assessments does not simply suggest a critical standpoint. While rejecting a quantitative representation of probability based on numerical probabilities, Keynes wanted to provide

structure for the use of qualitative, "non-numerical", as he called them, probabilities. Keynes's argument about probabilities that can be non-numerical, and thus may be difficult to compare, is introduced in the Part I of the book, devoted to the presentation of the "Fundamental Ideas" of his *Treatise*. However, Keynes also discussed at length a formal structure for comparisons between probability relations concerning those situations in which, while the standard approach to probability fails, there are still grounds on which qualitative probabilities can be of relevance and decisions can be "rationally" made. Part II of the TP states the "Fundamental Theorems" of an alternative probabilistic theory.

In Part II of the TP Keynes analysed how new probability comparisons, based on other comparisons constituting direct knowledge, some of which of numerical kind, can be obtained. And since numerical probabilities he considered as the exception, rather than the norm, he tried to establish the conditions for the ordering of probabilities in the case of non-numerical probabilities. The aim was to account for a comparison of relations of probability through the method of "numerical approximation", that he described as "the relating of probabilities, which are not themselves numerical, to probabilities, which are numerical", so that "numerical limits may be ascribed to probabilities which are not capable of numerical measures" (Keynes 1973 [1921], 132) [3].

Referring to George Boole's systematic method of approximation, interval-valued probabilities − indicating lower and upper bounds of a probabilistic representation of beliefs, instead of a precise number − emerge in Keynes's analysis as a possible representation of non-numerical probabilities (Brady and Arthmar 2012). In dealing with it, Keynes showed clear awareness that the incompleteness of the probability relation induces only a partial order: Keynes would have dubbed Savage's axiom P1 as too restrictive an assumption.[4]

This elaborate structure did not pass unnoticed at his time (Russell 1922), even because it did not remain simply a theoretical exercise. Keynes moved on in Part III to an analysis of induction based on it. And, before concluding with issues of statistical inference in Part V, he discussed what kind of application to decision-making his qualitative representation of probabilities would allow. Keynes was critical of G. E. Moore and of "normal ethical theory" because of its reliance on the mathematical expectation commonly used for games of chance. But Keynes's application of the

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[3] A passage, in which Darwin's caustic assessment of an opponent's arguments is used as exemplar of the fact that his method of approximation accords well with common discourse, is worth quoting: "Darwin was giving a numerical limit to a non-numerical probability when he said of a conversation with Lyell that he thought it no more likely that he should be right in nearly all points than that he should toss up a penny and get heads twenty times running" (Keynes 1973a [1921], 177).

[4] As already noted, a consistent set of axioms based on Keynes's intuition was provided only later, starting with Koopman (1940).In the statistical literature, the significance of this trend is testified by the work of Berger (1984) and Walley (1991).

theory of probability to "human conduct" goes further than a critique of maximisation of mathematical expectation, presenting the need for every sensible decision rule under uncertainty to incorporate a measure of the degree of confidence in the probability assessment. To this aspect Keynes devoted Chapter 26 of the TP on "The application of probability to conduct", a surprisingly rarely quoted chapter in the secondary literature.

Keynes (1973a [1921], p. 349) admitted that an alternative to the notion of mathematical expectations does not lie, in principle, "in the discovery of some more complicated function of the probability wherewith to compound the proposed good". But, he suggested nonetheless a formal example of how probability and the weight of argument might be compounded into a coefficient to be used in the shaping of a normative theory of decision-making. In his suggestion the original probability, once adjusted for the value of confidence in probability assessments (with the weigh ranging from 0 to 1), is transformed in what today we would call a decision weight in behavioural decision theory (Brady 1993), and on which a qualitative probability approach can be and has indeed been developed under the heading of non-additive probability theory (Basili and Zappia 2009).

For sure, such a reading of the TP possibly downplays a series of other relevant methodological aspects Keynes put forward. But concentrating on the formal backbone of Keynes's argument makes it possible to delineate Keynes's early approach to decision-making under uncertainty. One further main issue to consider, as a result, is whether this approach remains relevant even in the GT, where an aspect not accounted for in previous analyses, such as the conventional attitude of economic agents to try to forecast "the psychology of the market" instead of "the prospective yields of assets", does not certainly suggest the adherence to a rationalistic approach. There remains to consider, therefore, whether the analysis of the TP can be extended towards a proper theory of decision-making under the kind uncertainty dealt with in the GT.

But before moving on to this issue, it is worth noting that the analysis of the TP signals a crucial difference with Knight and his analysis of uncertainty. Knight did not aim at a theory of subjective probabilities of what he termed "estimates". When there is "no valid basis of any kind for classifying instances", the condition he saw for the definition of a priori and statistical probabilities, Knight (1964 [1921], 225) argued that "probability is involved in the greatest logical difficulties of all, and no very satisfactory discussion of it can be given". Henceforth, when discussing radical uncertainty one should distinguish Knightian uncertainty from Keynesian uncertainty (O'Donnell 2021). While for both Knight and Keynes uncertainty is irreducible to risk, unlike Knight, Keynes does have a theory of probability devised to deal with uncertainty as different from risk. This difference went lost in the development of mainstream decision theory. But while it can be argued

that Savage's systematization of expected utility theory addressed Knight, it is inaccurate to say that it addressed Keynes, a point first noted by Ellsberg (2001 [1962]).

6. The legacy of the *Treatise*

It has been argued in the previous section that Chapter 26 of the TP is the locus where to look for the application Keynes had in mind for his notion of probability. Indeed, it is in the concluding paragraph of that chapter that one can find the most direct indication of what probability was about for Keynes (1973a [1921], p. 356):

> The importance of probability can only be derived from the judgment that it is *rational* to be guided by it in action; and a practical dependence on it can only be justified by a judgement that in action we *ought* to take some account of it. It is for this reason that probability is to us the 'guide to life' [as Locke says].

Apart from issues of continuity or discontinuity between the TP and the GT, a crucial point to assess is whether the sentence just quoted may still have been in Keynes's mind while he was putting uncertainty at the centre of his inquiry. On this point, the correspondence with Hugh Townshend in the years just after the GT provides clear evidence for the fact that Keynes still had a keen interest in the role of probability in decision-making, and that the technical problem we have just seen he had left unsolved in the TP was still relevant to him. The definition of a criterion for decision-making alternative to what he termed "normal ethical theory" in the TP and identified with "strict mathematical calculation" used by Benthamite calculators in the GT appears to be central in his exchange with Townshend, an exchange that is characteristic for being the only one mentioning the TP in the correspondence related to the defence of the GT (Moggridge 1979).

While discussing with Townshend the characteristics of decision-making under uncertainty and in particular why he objected to the use of "equivalent certainties" to measure the outcome of decision taken under uncertainty, Keynes (1979 [1938], 288-289) writes:

> The matter you are tackling is a very important and interesting one often in my mind. … But a main point to which I would call your attention is that, on my theory of probability, the probabilities themselves, quite apart from their weight or value, are not numerical. So that, even apart from this particular point of weight, the substitution of a numerical measure needs discussion.

A comprehensive analysis of the correspondence shows that significant technical aspects of the TP survived Ramsey's critique (Zappia 2015). As noted by Townshend (1979 [1938], 292) the question posed by Keynes still was "whether … it may not be possible to develop a logical doctrine of equivalent certainties free from the assumption of numerical probabilities". Had he yielded to

Ramsey on the possibility to derive point probabilities from action in every instance, as the strict subjectivist perspective requires, Keynes would not refer to non-numerical probabilities as relevant objection to received analysis of decision-making under uncertainty.

In the 1938 assessment of his early beliefs, Keynes (1972 [1938]) appeared uncommitted to a defence of the epistemological foundations of his theory of probability (Davis 1994, Raffaelli 2006). He also may have been inclined, in the GT, to replace and explicit treatment of probability with his treatment of conventions (Dow 2003). Gillies and Ietto-Gillies (1991) suggest that Chapter 12 implicitly embraces a kind of intersubjective approach to probability. But the correspondence with Townshend shows that Keynes never stopped thinking of possible uses of his own 1921 theory of probability.[5]

Retrospectively, therefore, it can be argued that Keynes was still searching for an alternative to subjective expected utility theory well ahead of its precise definition. His 1937 insistence on uncertainty as the revolutionary element of the GT was not purely verbal. Keynes discussion of how alert agents (long run investors and policy makers) should behave when they deliberately choose not to trust the conventional judgement should be put in relation to his 1921 discussion of how dispose with normal ethical theory.

7. Concluding remarks

In the assessments of Keynes's theory it is usually contended that he was "hostile to exaggerate precision" and would favour a "a return to an economics that allows room for explanations of economic behaviour that cannot be expressed mathematically" (Skydelsky 2011, 12). A close reading of Keynes's works, though, indicates that while he rejected "strict" mathematical expectation and "calculable" probabilities, he did not object to mathematical methods and the use of probabilities in general (O'Donnell 1990, Backhouse 2010). His critique does not necessarily entail the endorsement of either irrationality or a theory of decision-making exclusively based on conventional valuations, such as that illustrated in Chapter 12 of the GT.

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[5] The 1938 exchange is also highly relevant since it is to Townshend that Keynes (1979 [1938], 293) presented the famous analogy that he was inclined to associate "risk premium with probability strictly speaking, and liquidity premium with what in my *Treatise on Probability* I called 'weight'" . The association of the liquidity premium with "an increased sense of comfort and confidence" cannot be accommodated within Ramsey's subjectivist perspective. Ramsey, de Finetti and Savage's strict Bayesian approach allows no room for a measure representing the degree of reliance on a probability assessment.

A rationalistic attitude that is proper of the TP also pervades most of the GT, and this hold true, it has been argued, even when uncertainty becomes the main issue of his analysis (Zappia 2016). The relevance of the TP 100 years after its publication stays, therefore, in the TP being the place where such a rationalistic attitude is first formulated by Keynes. Formally, it is endorsed in the kind of probabilities Keynes introduced: non-numerical probabilities as a representation for degrees of belief, conceived in such a way to avoid arguing that with non-numerical probabilities a valuation method of uncertain outcomes cannot be found.

This way of arguing is of relevance since markets of the kind discussed in Chapter 12 and the QJE essay typically presents neither situations of risk, nor situations in which there is absolutely no ground on which to take action. Keynes's rejection of the criterion for choice suggested by contemporary ethical theory and endorsed in economics through the application of "Benthamite calculation" preludes to the analysis of alternative criteria, normative rules to be followed by at least certain individual agents.

In conclusion, the paper suggests that the main enduring message of the TP is that it provides an *ante litteram* critique of mainstream decision theory that still is highly relevant.

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