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A Journey into Harold Hotelling's Economics†

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Harold Hotelling (1895–1973) was a major contributor to twentieth-century American economics. The overall thrust of his research, and his view of the role of mathematics in the discipline, have so far received little attention. Based on an unprecedented examination of his work and professional archives, this article provides a thorough analysis of Hotelling's background and contribution to economics. A self-taught economist in the 1920s, Hotelling built a research program that, despite apparently being highly technical, was primarily conceived as applied science to solve concrete social and economic issues, from spatial competition to natural resource exhaustion and public utility regulation. Although Hotelling's research was not exempt from criticism, it remains profoundly inspiring for the twenty-first century, from both a theoretical and epistemological point of view. When we remember that he trained the greatest, from Kenneth J. Arrow to William Vickrey, his career and ideas are all the more worthy of consideration. (*JEL* B21, B23, B31)

1. *Introduction*

Harold Hotelling's name belongs to the twentieth-century economists' hall of fame: everyone in the profession knows that they owe something—a result, a theorem, a rule—to Hotelling, in particular in fields such as welfare, spatial, and resource economics. The Hotelling rule describes the optimal allocation, over time, of exhaustible resources (Hotelling 1931b); the Hotelling law establishes a principle of minimum differentiation in competition theory (Hotelling 1929); the Hotelling lemma is commonly used for interrelated commodities in the theory of the firm (Hotelling 1932); the

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Hotelling T^2-test is an early result of econometrics (Hotelling 1931c); last but not least, Hotelling's welfare analysis and plea for marginal cost pricing (1938) gave rise to the marginal cost controversy (Frischmann and Hogendorn 2015) and through the concept of lump-sum taxes paved the way for the further development of the second theorem of welfare economics (Blaug 2007). Hotelling's papers are among the most cited from the pre–World War II economic literature, competing for instance with Ramsey's (1928) article on saving, von Neumann's (1928) proof of the minimax, Hicks's (1937) "Mr. Keynes and the 'Classics,'" or Samuelson's (1939) analysis of the multiplier.[1] Most remarkable is that each—not just one—of Hotelling's papers made an impact.

Economists and historians have so far paid little attention to Hotelling's general contribution to economic research. Fifty years after Hotelling's death, his ideas remain relatively unknown, and the literature rarely presents them comprehensively.[2] One reason for this may be the apparently scattered subjects of his contributions, belonging nowadays to well-separated subfields of economics. Another reason may be that, after intense research activity in mathematical economics and mathematical statistics from the 1920s to the end of the 1930s, Hotelling's publication activity significantly slowed down after 1946, when he moved from Columbia University to the University of North Carolina. There, he devoted his energy to the institutional and pedagogical promotion of mathematical statistics, leading tireless campaigns for the creation of dedicated institutes and departments around the world. Hotelling's career therefore developed partly against the current. He was a self-made pioneer of the use of a wide variety of mathematical tools in economics before the Second World War, and as such, was an exception in the American landscape. He was, however, no longer an active researcher when mathematical economics really took off in the United States. Some of his tools and concepts appeared old-fashioned (e.g., the monetary representation of utility and welfare) or quite outdated (the most recent optimal control theory offering wider applications than the classical calculus of variations), whereas some of his preferred themes would come to the forefront of economic research—general equilibrium analysis in the 1950s, resource economics in the 1970s, spatial economics in the 1980s. In the end, therefore, he was sidelined with respect to the very developments he himself had both called for and prepared for in his articles, as well as in the mathematical economics lectures he delivered in the 1930s.

This paper aims to provide a thorough, documented analysis of Hotelling's contributions to economics, from both theoretical and methodological viewpoints.[3] It is based on an analysis of Hotelling's published works and on a detailed examination of several archival collections from Columbia University (where Hotelling's papers are stored), the University of Washington (Seattle), and

[1] Data from Google Scholar (July 2024): 15,033 citations for Hotelling (1929), 8,095 citations for Hotelling (1931b), 2,532 for Hotelling (1931c), 1,596 for Hotelling (1938), 9,584 for Ramsey (1928), 4,139 for von Neumann (1928), 4,220 for Hicks (1937), 1,879 for Samuelson (1939).

[2] We have identified only a few works that have examined Hotelling's place in the history of economic thought. Darnell (1990b) devoted a volume to Hotelling's economic papers, with an insightful introduction on Hotelling's "life and economic thought," partly based on archival materials. Blaug (1992) mentioned Hotelling in his collection of "pioneers in economics." Hands and Mirowski (1998) presented Hotelling as a secondary character in the history of economic analysis, archetypal of a "neoclassical dream" (see also Mirowski 2002). Aydinonat and Köksal (2019) focused on Hotelling's location model as an interesting case of an explanatory model in the history of economics.

[3] Hotelling also contributed significantly to the field of statistics. Except where these contributions interface with his economic work, they have been left out of our analysis here. For details on Hotelling's contributions to statistics, see, e.g., Anderson (1960), Neyman (1960), Levene (1974), W. L. Smith (1978), and Darnell (1988).

Princeton University (where Hotelling was educated and started his professional career). In Hotelling's case the archival materials are particularly rich and allow his career to be reconstructed. They reveal meaningful, sometimes strategic choices of research and publications. They also offer an institutional and relational context that sheds light on his research agenda. Above all, they reveal the thematic and methodological mantras that framed Hotelling's economics.

Three main results emerge from our inquiry. First, Hotelling's mathematical economics, from his theory of depreciation (1925a) to his analysis of welfare (1938), was, despite being highly technical and despite the apparent absence of an empirical basis, conceived as applied science. Hotelling was not "a mathematician coming to economics" (Samuelson 1960) as Gérard Debreu was after him, for instance. He studied and used mathematics to solve concrete social and economic questions. Second, Hotelling's lifelong challenge was to convince his contemporaries that mathematics was a tool to reveal facts and the underpinnings of reasoning: if logical and field experiments were well designed, if models were properly built to reveal the logical and concrete implications of social representations and institutions, a logically cogent and transparent understanding of the economic world would result. Third, Hotelling's economics was oriented by a Georgist agenda, or at least by a constant preoccupation with comparing economic propositions in accordance with Henry George's legacy with other reformist recommendations. As we show below, bearing these three results in mind sheds new light on Hotelling's contributions to economic analysis.

The paper is organized as follows. Section 2 traces Hotelling's background and intellectual career from the 1910s to the 1960s, showing how he came to mathematical economics and what sort of issues he was confronted with in his early years. Section 3 turns to the role of mathematics in Hotelling's economics and in his wider research program in social sciences. Section 4 explores the relationship between Hotelling's theoretical contributions and concrete economic issues to be dealt with by policymakers, with special attention to his personal views on economic reforms. Concluding remarks follow.

2. *Hotelling's Career in Context*

2.1 *From "the Problems of the New City" to Mathematics*

Harold Hotelling was born in 1895, in the small rural town of Fulda, Minnesota. Describing his childhood, he liked to recount how his family, like many others, was confronted with the choice of leaving the pleasant routines of rural areas in order to find a more dynamic economic environment.[4] When he was nine, his parents decided to move to Seattle. There, his family found a "suddenly rising metropolis" experiencing industrial expansion, revenues from international trade, a rising population, and good public services and utilities.[5] The 1907 crisis partly dampened their hopes, impacting both Seattle's growth and Hotelling's family business projects. Access to good educational facilities, however, changed Harold Hotelling's life, as he recalled spending countless days in public libraries reading electricity books and "digging into many subjects, mainly scientific."[6]

[4] As a hay producer, his father was confronted in Fulda with harsh working conditions, the caprices of weather, and the "perfidy of the local agent of the railroad company," leading to daily concerns about rates and the irregular availability of freight cars (Western Hotelling and Allied Families, An Epic of Migration, 1948, Harold Hotelling Papers, hereafter cited as HHP, Box 52, "Printed Materials," 15).

[5] Ibid., 14.

[6] Ibid., 16.

In 1913, Hotelling entered the University of Washington for a bachelor's in journalism, with some interruptions in 1915–16 and 1918 to work as a journalist for a local newspaper and for military obligations. According to his autobiographical notes, his choice was motivated by "a long-standing and acute interest in problems of economic and political reform, stimulated by the democratic debates and problems of the new city."[7] In the 1910s, the political context was indeed abuzz. Woodrow Wilson created the Federal Trade Commission and passed the Clayton Antitrust Act in 1914; his economic advisor Louis Brandeis became a member of the Supreme Court in 1916, where he could enforce antitrust legislation (Berk 2009). And the general debate was embedded into progressive leitmotifs (Hays 1959, Leonard 2017).

In Seattle, Hotelling heard about the political debates of the time not only in the news but also in his daily student life. The University of Washington graduate school in political and social sciences was mostly influenced by the provocative reformist figure of J. Allen Smith. J. A. Smith (1907, 1914) aimed to demonstrate the antidemocratic intentions of the Fathers of the Constitution, or the way the Supreme Court had preserved the capital-owning class and corporate privileges against the rise of a true democratic organization. He was also interested in the relationship between the doctrine of laissez-faire and the rise of trusts and monopolies, investigating the sociological and political composition of the public utilities commissions that were flourishing throughout the country to organize the regulation of private and public utilities. Courses taught at the University of Washington were oriented toward the functioning of economic institutions—monetary systems, markets, and Supreme Court decisions in relation to economic issues. In this environment, Hotelling found clues to understanding the global transformations of the US economy that echoed his own story: cartels and their high prices, population migrations, the difficulties of regulating access to and prices of public utilities.

The journalism curriculum emphasized political science and political economy, but also business administration and accounting, including asset theory. Hotelling concluded that the best way for him to contribute to the debates of his time was to switch to economics, which implied, in his mind, improving his skills in mathematics:

> The formal study of several branches of economics while I was nominally a student of journalism laid an invaluable foundation for later work. […] The combination of science and political economy led to the thought of applying the methods proven so fruitful in the exact sciences to discover new truth in economics and political science. Proficiency in these methods required in the first place mathematics.[8]

When Hotelling returned to the University of Washington in 1920, it was therefore to begin a master's program in science, with applied mathematics as the main subject (1920–21).[9] He studied analytical mechanics with Eric Temple Bell, differential equations with Lewis J. Neikirk, and advanced mathematical finance and "mathematical theories of investment" with Robert E. Moritz.[10] Under Neikirk's supervision, he wrote his

[7] Ibid., 17.

[8] Ibid., 17.

[9] Application form for admission to the Princeton graduate program, February 21, 1921, Princeton University Archives (hereafter cited as PUA), Box 36, "Hotelling Harold."

[10] Moritz, head of the mathematics and astronomy department, encouraged "more scientific" preparation for students in economics and commerce, through the introduction of mathematics and statistics in economics departments (Moritz 1919). His course dealt with "the application of algebra to problems of compound-interest, annuities, amortization, bonds, sinking funds, depreciation, and building and loan associations." Catalog for 1920–21: http://www.washington.edu/students/gencat/archive/GenCat1920-22v1.pdf.

master's thesis on the dynamics of migration, showing that adopting a "macroscopic view" allowed Fourier's law of heat conduction to be applied to population movements (Hotelling 1921). Hotelling then moved to Princeton University's department of mathematics, in September 1921, hoping to study mathematical economics or mathematical statistics, realizing, however, that no one there "knew anything about the two subjects."[11] He studied analysis, mathematical physics, and astrophysics, but also, and more deeply, topology (analysis situs) and differential geometry: his PhD thesis, supervised by Oswald Veblen and James Wadell Alexander, explored the properties and classifications of three-dimensional figures invariant under continuous algebraic transformations (Hotelling 1925b).

In March 1924, when he moved to Stanford's Food Research Institute, Hotelling was therefore armed with several kinds of mathematical tools and able to choose those he considered most appropriate to the specific subjects he was dealing with. This gave him a freedom unavailable to earlier mathematical economists or even to his contemporaries—the few economists using mathematical tools, such as Irving Fisher, Griffith C. Evans, and Charles F. Roos, were indeed trained mainly in arithmetic and differential calculus (Weintraub 2002).

2.2 *The Stanford Years: A Time of Intense Activity*

Innovative ways to mobilize mathematics, in particular in economics, were precisely what Carl L. Alsberg and Ray L. Wilbur were seeking when recruiting a young mathematician at Stanford. The position was created with the ambition of adding someone to the team who would be able to bypass the technical difficulties encountered in the "crop estimating project," in particular in the analysis of "correlations between climatic factors and crop yields" but also in the development of "statistical studies having to do with price index numbers, the evaluation of consumptive demand and the like."[12] The objective was also to bring fresh knowledge in pure mathematics to the institute, and the fellowship contract authorized large time slots for personal projects—a freedom that was quite uncommon at Stanford, compared to other departments (Cherrier and Saïdi 2020). Hotelling wrote many of his most important papers during his time at Stanford, first at the Food Research Institute (1924–27) as an associate researcher, then in the mathematics department (1927–31) as an associate professor of mathematics.

He opened two research areas. On the one hand, his contribution to crop estimation led him to develop a research program in mathematical statistics. While helping the staff with technicalities, he decided to make an "attack upon the foundations of statistical theory."[13] Following Alsberg's and Conrad P. Wright's advice,[14] he discovered Ronald A. Fisher's experimental program (1925). He undertook both to make Fisher's work known in the United States and to confer upon it a sound logical background—a life-long research program that began with Hotelling's paper on the generalization of the Pearsonian correlation coefficient (1925c). Hotelling introduced a seminar on the mathematical theory of statistics at Stanford, where he started

[11] Autobiographical Remarks, 1963, HHP, Box 6, "Wallis-Fry," 4. Hotelling's fellow mathematician Albert Tucker (1985, p. 4) recalled in an interview that Hotelling mistakenly expected Thorstein Veblen to be his advisor, and only later realized that it would be Oswald Veblen instead, and therefore that mathematics rather than economics would become his main field of expertise.

[12] Alsberg to Hotelling, 13 March 1924, HHP, Box 6, "Wallis-Fry."

[13] Hotelling to Bennett and Wright, 30 January 1925, HHP, Box 41, "Agriculture III."

[14] Hotelling to Calkins, 22 August 1944, HHP, Box 2, "Miscellaneous C."

with probability, which was not common at the time among statisticians (Morgan 1990), and ended with his own work on correlations (Carvajalino and Mueller 2024). A few months after Hotelling's arrival, Holbrook Working was recruited at the Food Research Institute. Both individuals shared strong enthusiasm for R. A. Fisher's work. They later coauthored a paper on multi-correlations (Working and Hotelling 1929) and exchanged ideas on economic issues such as expectations in price formation and stability conditions.[15]

On the other hand, personal research brought Hotelling back to mathematical economics. Abandoning the macroscopic view he used in his study of migration—and the mechanical analogy with heat conduction laws—this time he chose a microscopic perspective to establish "a mathematical theory of depreciation," combining the economic principle of rational choice and the selection of accurate mathematical tools, that is, functional analysis (Hotelling 1925a). The paper provided the occasion to tackle a topical issue and was decisive for Hotelling's later path in mathematical economics.

Depreciation issues were indeed widely discussed in relation to taxation and the fair pricing of public utilities (Allison 1914). The question was whether depreciation should be considered as a production cost or as an erosion of capital value. In the first case, it was legitimate to include depreciation in the sales price; in the second, the private or public capital owners had to admit that depreciation lowered their profits. The Supreme Court had recognized the practice of including depreciation allowances in the sales price of public utilities with *City of Knoxville v. Knoxville Water Co.*, 212 US S.1, 1909, but the decision was still controversial, and the American Society of Civil Engineers (ASCE) had asked for further studies. In its final report (Stearns et al. 1917) the ASCE challenged the way depreciation methods were used, stimulating many contributions (e.g., Taylor 1923, Skinner 1924, Canning 1929, see also Giocoli 2018).

Hotelling had been in contact with the subject during his bachelor curriculum, his master's in applied science, and on his arrival at the Food Research Institute, where he discussed the subject with John B. Canning.[16] He wrote the paper quickly, in October and November 1924, defending an approach challenging the Supreme Court's decision. Rather than considering depreciation as wear and tear of productive capital (machines), as in unit cost theories of depreciation (Taylor 1923), Hotelling (1925a, p. 343) considered the issue from the angle of the economic value of the machine perceived as a "productive asset," an actualized sum of expected future profits in continuous time.

Archival materials show that Hotelling immediately saw the potential of his analysis for the study of another kind of productive asset, "exhaustible assets," a subject he also described as "mining economics."[17] Some preliminary results, dealing with "competition of exhaustible assets" and "monopoly of exhaustible assets" were presented at the Chicago Meeting of the American Mathematical Society on December 26, 1924.[18] Important issues, such as whether depletion should be considered an erosion of the value of the mine or whether coal prices were fair ones, entered Hotelling's agenda as

[15] Effects of Uncertain Forecasts on Prices and Stocks, March 1927, HHP, Box 41, "Agriculture III."

[16] In the submission letter accompanying his paper to the *Journal of the American Statistical Association*, dated December 19, 1924, Hotelling indicated that Canning checked "the non-mathematical part" of the paper. Hotelling to Ogburn, December 19, 1924, HHP, Box 10, "AMS Reports and Correspondence (3)."

[17] HHP, Box 41, "Agriculture III." See also Darnell (1990b).

[18] HHP, Box 10, "AMS Reports and Correspondence (3)."

he read Saliers's (1922) accounting book on depreciation.[19]

The subject was, however, more challenging than depreciation. First, because Hotelling needed a clear mathematical representation of competition and monopoly, something he did not have at his disposal in 1924–25. Second, because the case of exhaustible assets required more complex mathematics, in particular the calculus of variations, "this rather abstruse branch of mathematics . . . necessary . . . for dealing with such problems as the determination of the most profitable rate of working a mine when demand is elastic."[20] Third, because Hotelling also wanted to study the concrete functioning and constraints of the mining industry, in parallel with his theoretical endeavor (Ferreira da Cunha and Missemer 2020).

Drafts and notes from 1925 show that, for a few months, Hotelling struggled with the definition of competition. His objective was to "study by means of the same analysis monopoly and competition, usually theoretically treated as distinct problems, but actually shading gradually into each other."[21] The staggered exhaustion of resources of competing mines was indeed a logical example of the continuity between monopoly and competition: competition could turn into oligopoly, duopoly, or monopoly if resources were progressively exhausted for a competitor.[22] However, logical problems appeared when demand ceased to be inelastic in the case of a monopoly.[23]

In January 1926, Hotelling decided to focus on writing a "theory of competition," gathering notes he had accumulated over the previous years.[24] His research helped him improve his knowledge of the economic literature, discovering in particular Moore's (1906) "Paradoxes of Competition" and above all the three volumes of Edgeworth's papers (1925a). Special attention was given to duopoly and criticism of Augustin Cournot, Joseph Bertrand, Alfred Marshall and Edgeworth, in particular their instability results, due to their "neglect of an important feature of real markets, namely, that most buyers are in positions where it is more advantageous to make a particular purchase from one seller than any other."[25]

During the next few years, Hotelling worked simultaneously on three different mathematical economics projects. The paper on stability in competition became an investigation of duopoly and local differentiation, taking its final shape in July 1928. The work on exhaustible assets became a 14-section study of optimal paths of depletion of exhaustible resources. Based on partial equilibrium, it explored various configurations—free competition, private or public monopoly, with or without extraction costs, with or without stock effects—with different objective functions—maximization of profits or maximization of the social value taken from the resource. The last project was even more ambitious, connected

19 On Hotelling's sources in accounting for his research on exhaustible resources, see Missemer, Gaspard, and Ferreira da Cunha (2022). Hotelling was apparently unaware of the existing economic literature on exhaustible resources and conservation (e.g., Gray 1913, 1914; Ely 1918; see G. A. Smith 1982; Ramos Gorostiza 2003; Missemer 2017) when he began his work on exhaustible assets.

20 Monthly report to the directors of the FRI, 1 December 1924, HHP, Box 41, "Agriculture III."

21 HHP, Box 42, "Exploitation of Irreplaceable Assets."

22 In the first drafts from December 1924, competition was represented as a system of price-setting producers within an industry, as in Cournot. Monopoly appeared as a special case in which the number of producers was equal to one. In a second series of drafts, dated April 1925, Hotelling tried to solve an intertemporal equilibrium with n producers facing a common (elastic) demand function with different (unknown) stocks of exhaustible resources. He finally abandoned this direction in favor of a partial equilibrium framework with price-taking producers.

23 Two mines, inelastic demand, March 1925, HHP, Box 42, "Exploitation of Irreplaceable Assets."

24 Monthly report to the directors of the FRI, 2 February 1926, HHP Box 41, "Agriculture I, II, III, IV."

25 Ibid.

to a more general problem of economics: the mathematical representation of "joint supply and demand" with interrelated commodities, a subject he soon linked to an analysis of Edgeworth's taxation paradox (1897, 1925b). This states that a unit tax imposed on one of two commodities produced by the same monopolist can be associated with lower prices.

Already in February 1926, Hotelling had hoped that using "the methods of differential geometry" would help extend and generalize some results of economic theory—as developed, for instance, in Irving Fisher's (1892) doctoral thesis—and even to "overthrow or seriously modify a good deal of economic theory," allowing economists to get away from an assumption that they were compelled to make, "that the utility of every commodity is independent of the quantities of other commodities."[26] The intuition was that it was possible to consider sets of (n) commodities as coordinates of a point in n-dimensional spaces, and to apply to them different kinds of algebraic transformations (translations, rotations), vector calculus, or, more simply, functional calculus. In particular, considering utility and profit functions as such transformations could reveal which of economics' well-known results would withstand generalization, or whether some enigmatic results could be explained by a more systematic (and mathematical) inquiry. Just as physics had experienced its revolution thanks to Albert Einstein and Hermann Weyl's tensor analysis, economics would be profoundly modified by differential geometry. What appeared to be a mere curiosity like Edgeworth's paradox could, for instance, prove to be more frequent than expected.

As early as May 1926, Hotelling was puzzled by Edgeworth's paradox, searching for numerical configurations that supported it.[27] Was the result due to the monopolist's behavior (and to the nature of supply) or to the nature of demand(s) when interrelated commodities were at stake? Once again, it took several years to answer the question, requiring both the mathematical developments mentioned above and the establishment of systematic case studies on the application of taxation to supply and demand.[28]

This project was paused for a while by a major piece of work in mathematical statistics and by the organization of a trip to Britain with the aim of meeting R. A. Fisher at the Rothamsted Experimental Station and, if possible, to visit "Cambridge and other School places, and to meet [John] Wishart, [Arthur] Bowley, [John Maynard] Keynes, the Biometric School people, F. P. Ramsey, and others."[29] Hotelling's last year in Stanford was mainly occupied with collecting data on demand and supply, by writing a paper on the causes of birth rate fluctuations, and by several reviews that stimulated reflection on the role of mathematics in science.

2.3 *Columbia: General Welfare*

In 1931, when Hotelling started his professorship in economics at Columbia University, three of his main contributions to mathematical economics were already

[26] Monthly report to the directors of the FRI, 2 February 1926, HHP, Box 41, "Agriculture I, II, III, IV."

[27] Details of calculation of Edgeworth's paradox, May 1926, HHP, Box 39, "Misc. (5)."

[28] Hotelling could not prove that the paradox had a high chance of happening in real cases, but he could at least prove that it was more than a mere curiosity. He provided examples with free competition and monopoly, and even a set of conditions that would have allowed the generation of as many examples as desired. Lowering prices by levying a tax would have been like having your cake and eating it too: it would have both lowered monopolists' gains, their rent, increased the public budget, and by lowering prices, increased social welfare.

[29] Hotelling to R. A. Fisher, April 28, 1929, HHP, Box 14, "IMS Draft for National Roster." Finally, Hotelling went to Britain only for a few months, from the end of June to the end of December 1929. There is no evidence in the archives that he met all the British economists he listed (Gaspard and Missemer 2019).

achieved (1929, 1931b, 1932). Strong institutional backing allowed him to campaign for the rise of mathematical economics and mathematical statistics. He spent considerable time promoting and running old institutions such as the American Statistical Association (serving as vice president in 1941), the American Mathematical Society (as a member of the council), the Royal Economic Society (as a member), and the American Association for the Advancement of Science (as vice president for social and economic sciences in 1942). He was also an active participant in emerging organizations such as the Econometric Society (as president in 1936 and 1937) and the Institute of Mathematical Statistics (as chairman of the nomination committee in the late 1930s and as president in 1942).

At Columbia, Hotelling became an influential professor: from 1931 to 1944, approximately 80 students followed his classes in mathematical economics, among them Kenneth J. Arrow, Robert Dorfman, Solomon Fabricant, Milton Friedman, William Madow, William Pabst, Gabriel Preinreich and William Vickrey.[30] Hotelling also started his "Hotelling Teas," informal meetings at home. At the beginning of the 1930s, he taught not only his own economics but also that of classic authors in mathematical economics (Cournot, Léon Walras, Vilfredo Pareto, Bowley, Jules Dupuit). His knowledge of the literature had improved, which allowed him to specify the characteristics of his own research, compared to Roos's, Evans's, or Roy G. D. Allen's for instance (e.g., Hotelling 1931a, 1935, 1939; see Darnell 1990a).

The Great Depression did not fundamentally change Hotelling's main preoccupations in economics, offering, however, a new and decisive occasion to develop his views of competition structures. He rejected the explanations of the crisis in terms of overproduction, pointing out the overwhelming power of cartels and private monopolies. The latter captured the benefits of technical progress—hindering both researchers and consumers from taking advantage of innovation[31]—and set prices so high that they hindered any increase in demand. In December 1933, at the annual meeting of the Econometric Society in Philadelphia, he vigorously attacked Roosevelt's New Deal, in particular the priority given to reflation through the Agricultural Adjustment Act (May 12, 1933) and the National Industrial Recovery Act (June 1933), which reintroduced the legality of cartels:

> The success of the government's recovery program. . . must be judged, not in terms of price levels, but in terms of the quantity of physical goods and services which are put into the hands of consumers. With this is to be considered the effect of the program on the distribution of wealth among different classes. But the chief thing needed is to increase physical production. In this respect, what is being done at Washington is definitely in the wrong direction. The attempts to increase the prices and curtail the production of oil, agricultural products, and other commodities are anti-social.[32]

Hotelling denounced the fact that recovery plans would rest on (hidden) rising inequalities, each industry maximizing its own income rather than promoting collective welfare. He furthermore noted that, after four years of depression, some prices had

[30] A fairly complete list of Hotelling's students can be found at: http://www.irwincollier.com/columbia-economics-mathematical-economics-hotelling-class-rolls-1931-1944/.

[31] 'Research and Obsolescence—Profit and Loss," Address to the American Chemical Society, June 1932, HHP, Box 46, "Lectures by Hotelling: Econ. of Obsolescence...." The address suggests that a mathematical analysis could define an optimal rate of investment (i.e., research and development) in new products and new production methods to avoid too rapid obsolescence.

[32] Prosperity Through Increased Production, Dec. 1933, HHP, Box 25, "The General Welfare to 1938," 1.

been maintained high relative to others—in particular transportation and energy prices. He proposed his own plan consisting rather of drastically lowering transportation and distribution margins—most notably bridge tolls and railroad freight rates—either by the introduction of more competitive practices or by public ownership of utilities in the case of decreasing production costs: in such cases indeed, laissez-faire would naturally lead to monopolistic structures. Public ownership, seeking to maximize public interest and not personal profits, would allow the introduction of fair prices.

Hotelling sketched a mathematical proof of the intuition that in the case of decreasing cost industries, the operating revenue (i.e., utility rates) should not be equal to overheads and dividends. Collective welfare would be maximized by operating without constraint or by operating to the marginal cost of production:

> I would not suggest that the fullest prosperity is to be achieved by an attack on railroad problem alone. But the establishment of a new deal … might well begin with the railroads, from which it could proceed to the electric, gas and telephone utilities, and then to the other large industries in which marginal cost is at present only a small part of the price paid by consumers. … [W]e are in a large depression, from which we are not likely to emerge until after measures are taken of a far different and more drastic character than any which now figure in the news dispatches from Washington.[33]

Parts of the argument were finally published, with a polemical tone, in the columns of the *Columbia Alumni News* (Hotelling 1936a). The results were more formally defended at another meeting of the Econometric Society, in December 1937, under the title "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates."[34] From December 1933 to December 1937, Hotelling accumulated examples of the impact of excise taxes and formal proofs of what finally became his theorem. "[T]he maximum of general welfare" would be obtained if each industry was able to sell at marginal cost (Hotelling 1938, p. 253). As far as parts of railroad rates or bridge tolls were to be considered as excise taxes, they had to be avoided.

Archival materials therefore reveal the story behind Hotelling (1938). The formal proof it contains corresponds to intuitions he had been tirelessly expressing since the middle of the 1920s, that free competition would maximize collective welfare and provide optimal and fair prices while excise taxes, tolls, and depreciation charges would hinder such a result. Hotelling clearly considered, however, that free competition was an abstraction and did not exist in the real world—all his papers from the period are explicit on this matter. Large parts of the industry sought to establish stable monopolies or oligopolies and capture important parts of the value. Overwhelming prices—parts of the prices beyond the marginal cost—generated social inefficiency and net welfare losses, as did excise taxes. That was why public regulation was often needed. In Hotelling's view, governments had to operate the railroads and utilities and adopt taxes on income, inheritance, and the site value of land (1938, p. 242). All this would not have been established without advanced mathematical tools, allowing intertemporal optimal pricing and supply and demand functions of related commodities to be described.

Apart from his criticism of I. Fisher's proposal to reform income tax by exempting savings (Hotelling 1943), the 1938 paper was Hotelling's last major publication in economics. Nonetheless, the archives show that he never stopped working on economic topics—principally following on from his main publications—and that he continued to read and comment upon the economic literature.

33 Ibid., 21–22.

34 HHP, Box 25, "The General Welfare."

Some examples of these late reflections are given in the next sections.

From the 1940s onward, Hotelling devoted most of his energy to mathematical statistics, both in terms of scientific contributions and in terms of institutional responsibilities. During the war, he led the Statistical Research Group composed of Abraham Wald, Allen Wallis, George Stigler, and Milton Friedman, among others. He lobbied for the creation of an ambitious statistical department at Columbia but did not obtain the support he asked for, only getting the recommendation for graduate students to follow courses in mathematics and statistics. In January 1942, the university published (as internal documentation and in the *New York Times*) the following communiqué:

> Mathematical Preparation: The use of mathematics, including higher mathematics, has become important in several branches of economics and statistics. Much of the recent literature of general economics is written in a language not easily understood without some knowledge of the differential and integral calculus. Students planning to work for the Ph.D. degree in Economics will therefore find it advantageous to acquire familiarity with the calculus and higher algebra before beginning their graduate studies in Economics. (Columbia University 1942, p. 18)

Hotelling finally left Columbia for the University of North Carolina at Chapel Hill in 1946. Frank P. Graham and Gertrude Cox gave him the opportunity to establish an independent department of mathematical statistics there, presumably the first department specializing in the field in the United States (Neyman 1960, Agresti and Meng 2013). With total autonomy over teaching content and recruitment, Hotelling served as chair of the department until 1952, when he became associate director of the Institute of Statistics. He ended his career as a Kenan Distinguished Professor of Statistics. Until his retirement in 1966, he continued to promote the implementation of quantitative methods in social science departments, not only in the United States but also elsewhere, in particular India (W. L. Smith 1978, Arrow and Lehmann 2005). He remained quite active until a sudden decline in his health in 1972; he died in December 1973.

From journalism to mathematics, economics, and statistics, Hotelling covered extended fields of research throughout his career, leaving each time, for the last two fields at least, an indelible trace in history.

3. *Mathematics as "the Most Fundamental Subject of All"*

3.1 *A Universal Toolbox for the Social Sciences*

It is clear enough that Hotelling had a double background: a strong interest in political, economic, and social issues right from the 1910s through his curriculum and early professional experiences in journalism; and advanced skills as a Princeton-trained mathematician. This combination profoundly structured his agenda, not only in economics but also in statistics and in his conversations with psychologists, sociologists, and political scientists. As mentioned above, one of the reasons why he was appointed to Stanford's Food Research Institute in 1924 was his double interest in advanced mathematics and social and economic issues. We can also suspect that the reason why Wesley C. Mitchell asked him in early 1931 to join Columbia University—an institutionalist place at the time (Rutherford 2004, 2011)—was that Hotelling was not a pure mathematician peripherally involved in economics, but a researcher deeply interested in making his mathematical skills useful to the social sciences.

Hotelling's PhD at Princeton deeply framed his understanding of what applying mathematics meant (Gaspard and Mueller

2021). Before his arrival, he considered the application of mathematics as a transfer of equations and formalisms from physics to the social sciences. At Princeton he met with O. Veblen and Alexander, two architects of the "American Postulate Theory" (Scanlan 1991), and inherited from that period a new representation of the articulation among logic, mathematics, and sciences. In O. Veblen's understanding, topology was considered to be the basis of every branch of scientific knowledge, and by adding axioms, one could design analytical frameworks suitable to less general mathematical fields, with physics at the very end. In Hotelling's mind, other restrictions could be chosen to develop different mathematical interpretations corresponding to other fields—or subfields—of knowledge. After his PhD, he explored the possibility that mathematics could be used as a toolbox, applicable to multiple kinds of problems, and that specific mathematical instruments could be developed for the social sciences. As he would write in the mid-1930s, "there is nothing that has a richer profusion of applications, there is nothing that travels over the whole domain of human knowledge as does mathematics" (Hotelling 1936b, p. 158).

From Hotelling's perspective, mathematics covered a wide range of tools, not only "equations, but […] also graphs, models, and other aids to reasoning."[35] What mattered the most, however, were not all kinds of mathematics, but the advanced, "higher" tools, including for instance (in the 1940s–50s) "excerpts from the theory of knots, properties of prime numbers, and curvature properties of surface."[36] Using several new tools was the best way of obtaining solutions to complex problems for which usual reasoning would prove insufficient, and even for which past mathematical proofs (founded on rudimentary mathematics, i.e., the old mathematics of physics) would prove erroneous. In a draft on the teaching of mathematics, echoing his research on Edgeworth's paradox, Hotelling gave the example of "the effects on social welfare of systems of taxation of interrelated commodities" for which "advanced calculus [was] necessary." The evaluation of Dupuit's and Marshall's classical results on taxation in the case of interrelated commodities required differential geometry.[37] The same applied, for example, to his economics of exhaustible resources, for which he argued that "problems of exhaustible assets [could not] avoid the calculus of variations, including even the most recent researches in this branch of mathematics" (1931b, p. 140). As soon as socially and economically complex issues arose, advanced mathematics was not an option among others, rather a requirement.

In Hotelling's research program, statistics and economics were advanced mathematics' main fields of application. Yet he regularly pointed out that many other disciplines would benefit from such tools. Obviously, this was true first of all for physics and biology, as testified by a talk given by Hotelling in October 1931 on the applications of differential geometry.[38] But it was also true for the social sciences (not only economics) and humanities, including literature, where "counts of certain combinations of words in the writings" could help identify the author of an anonymous poem (1936b, p. 168).

Besides economics, the archives show that particular attention was paid to political science, sociology, and psychology. As for the former, we know that Hotelling kept

[35] The Scope of Mathematics in Economic Theory, undated (1930?), HHP, Box 46, "Lectures by Hotelling."

[36] Future Research Workers Need Mathematics Now, undated (1940s–50s?), HHP, Box 25, "Future Research Workers Need Maths Now."

[37] Ibid.

[38] Talk Before Mathematics Colloquium, 27 October 1931, HHP, Box 44, "Spaces of Statistical Parameter."

a constant interest in political and electoral issues, right from his early, student-era inquiries in Seattle in the 1910s at least until his article on the two-party system (Hotelling 1950). Presumably at the beginning of the 1930s, he drafted a short note on the "applications of mathematics to political science," containing references to previous works in the literature using algebra, complex geometry, statistics, and least squares to analyze voting systems and the "apportionment of representatives."[39] With respect to sociology, Hotelling gave a lecture in Seattle in the 1930s where he stated: "sociology [includes] sample surveys—a branch of the theory of experimental design—[and] populations [issues, including] fitting by logistic curve."[40] Finally, regarding psychology, in a letter to psychologist Robert R. Holt dated April 7, 1951, he reviewed the different applications of mathematics and statistics that he considered insightful for the field, such as "matrix theory, probability, and theory of estimation and testing, analysis of variance, and particularly multivariate analysis."[41] In each case, social scientists had a lot to gain from improving their knowledge of mathematics.

In a sense, Hotelling considered mathematics to be the gospel for all disciplines, that is, a common set of tools revealing new results in all bodies of knowledge. It could serve "everywhere"[42] because it did not have the same status as other disciplines, as "the most fundamental subject of all"[43] with an "extreme degree of […] generality" (Hotelling 1936b, p. 157).

Considering mathematics as universal did not imply that the same mathematics was to be used in the physical and social sciences, and in each field of application in particular. It is interesting in this regard to observe that Hotelling did not use his abilities in topology to address economic issues, while others would do so from the 1940s onwards. In retrospect, this might appear as a limitation on Hotelling's contributions to mathematical economics but it should also be kept in mind that, from his perspective, not all sorts of mathematics necessarily suited all disciplines. While preparing a talk in October 1930, Hotelling drafted a list of fields with corresponding primary mathematical tools to be employed: statistical mechanics and matrix theory for physics, analysis situs for astronomy, calculus of variations for economics, symbolic logic and probability for political science, Brownian movement for biology, etc.[44] Mathematics provided a wide array of tools but each discipline, and moreover each subdiscipline, had its specificities, and therefore its most suitable tools.

3.2 *The Role of Mathematics in Economics*

In Hotelling's view, economics was still, like many social sciences, in its infancy. Economists' discourses often contained unproven assertions, such as the efficiency of laissez-faire and the alleged universality of the law of supply and demand. They also often rested upon common-sense ideas such as the need to tax mining profits for resource conservation (Hotelling 1931b) or to price public utilities such that the full cost is recovered (Hotelling 1938), while these ideas were revealed to be false when correctly examined, especially by mathematical reasoning. Some progress had

[39] Applications of Mathematics to Political Science, undated (early 1930s?), HHP, Box 45, "Misc. Problems, Undated (1)." Perhaps this is one of the origins of Arrow's interest in the voting paradox, as Arrow was one of Hotelling's students at Columbia.

[40] Lecture in Seattle, "Mathematical Statistics for Economics and Sociology," undated (1930s?), HHP, Box 46, "Lectures by Hotelling."

[41] Hotelling to Holt, 7 April 1951, HHP, Box 3, "Miscellaneous H."

[42] Fifteen-minute Talk to Math. Club, 23 October 1930, HHP, Box 8, "Memos."

[43] Hotelling to Alter, February 20, 1938, HHP, Box 9, "American Association of University Professors."

[44] Fifteen-minute Talk to…, op.cit.

been made thanks to Cournot, Edgeworth, Walras, and I. Fisher, but even their results could be reassessed thanks to new kinds of mathematics, such as integral calculus, and more generally n-dimensional analysis. For instance, considering a market not as a point but an "area" changed the representation of competition (Hotelling 1929). And the use of integral calculus to compare time-deploying surplus or utilities revealed that private monopolies (under suitable taxation policies) would exhaust resources slower than public ones (Hotelling 1931b). Differential geometry would offer integrability conditions unattainable without it (Hotelling 1932).

Mathematics was, however, more than a heuristic tool for Hotelling. He considered it an unequivocal, transparent language, allowing the passage from premises to conclusions without adding anything to or removing anything from the content of premises. Such a feature had two major virtues for economics.

First, mathematics could help elucidate the tacit assumptions behind common beliefs, or even behind theories and models: "the object of a math. proof is not merely to make the result plausible, but to show how it follows from the premises and thus to throw light on the premises."[45] Mathematics required a system of hypotheses from which it was possible to prove virtually anything. The point was not to demonstrate a result, but rather to shed light on the hypotheses on which the demonstration was based; when they proved excessively restrictive or far-fetched, they tended to make what they demonstrated equally restrictive or far-fetched:

> There is … a proposition, in which almost everybody believes, that if everyone is left to himself and will just pursue vigorously his own maximum profit, then everybody will be as well off as possible. . . . I make a practice each year of presenting to my course in mathematical economics a mathematical demonstration of this proposition of *laissez-faire* as nearly as I can formulate it mathematically. The point in that is not to make people believe in the proposition—they believe in it any way—but to show what definitions and what assumptions have to be made in order to make a mathematical proof possible. By the time a person has understood the definitions and the assumptions involved in these proofs, he is quite willing to reject the result. (Hotelling 1936b, p. 163)

Second, mathematics had the characteristic of being value neutral: contrary to ordinary language, it neither introduced nor strengthened normative contents—a feature that was essential for making economics a science. From Hotelling's perspective, it functioned like a catalyzer in chemistry having no impact on the nature of the component entering the chemical reaction. Models were used to make comparisons between alternative industrial organizations (e.g., free competition *versus* monopoly), alternative political decisions (e.g., excise taxes *versus* severance taxes), or even alternative forms of social organization (e.g., search for individual profit *versus* search for collective good). Ideally, comparison terms (quantities, prices and welfare) had to be as neutral as mathematics itself. This is why Hotelling constantly rejected subjective representations of utility, wishing during his entire career to convince his fellow economists that aggregate monetary surplus (producers' profits and consumers' personal profits) were more suitable indicators than welfare valuations (Gaspard and Mueller 2021).

4. *Economics as a Science for Policymaking*

4.1 *Theoretical Abstraction and Empirical Reality*

In line with the institutionalist mantra of the 1920s and 1930s (Rutherford 2011, Hédoin 2013), according to which economic research had to be both theoretically relevant and empirically grounded in order

[45] The Scope of Mathematics…, op. cit.

to address concrete problems, Hotelling considered that advanced science required "interactions of inductive and deductive processes" with constant to-ing and fro-ing between empirical facts and mathematical theorization.[46] "Preliminary classification of facts" helped early forms of formalization enabling, in return, the identification of "further types of facts which [came] gradually to influence more and more the direction and purpose of research in the science."[47] Behind the apparently high degree of abstraction of his theoretical contributions, he thus showed profound concern for empirics, real-world issues and, as astonishing as it may seem, realism.

It has been shown, for instance, that Hotelling's exhaustible resource economics (1931b) entertained a complex relationship with empirical realities. The Hotelling rule that we usually retain from the paper, that is, the principle according to which the price of an exhaustible resource should grow at the pace of the interest rate, has been proved to be hardly observable in historical data (Halvorsen and Smith 1991, Livernois 2009, Slade and Thille 2009, Hart and Spiro 2011, Gaugler 2015, Karp 2017), except maybe for old-growth timber under specific circumstances (Livernois, Thille, and Zhang 2006). The archives reveal that, actually, Hotelling did not consider the rule as a good description of price trajectories. He initially had two research leads in mind: a generic project, a development of his theory of depreciation (1925a) on the intertemporal use of what we may call *pure irreplaceable assets*, that is, assets whose sole characteristic would be to be exhaustible; and a more concrete, policy-oriented project on natural resources such as fossil fuels, for which other features (e.g., geological constraints, natural monopoly) had to be taken into account. While the published version of the 1931 article mostly exhibits Hotelling's results on the former project, his unpublished drafts and notes show his behind-the-scenes interest in the concrete functioning of energy markets—they contain, for instance, a letter from petroleum engineer Stanley C. Herold, dated July 7, 1930, answering Hotelling who had requested information on oil industry extraction costs.[48] Traces of this more policy-oriented project can be detected from section 8 onward of the final 1931 paper, but readers may not pay attention to them if they are not aware of Hotelling's preparatory materials revealed by the archives. This double reading of Hotelling (1931b) has some importance insofar as it sheds new light on the validity of the Hotelling rule (Ferreira da Cunha and Missemer 2020) and on the way Hotelling conceived empirical applications of his model, also in relation to the role of the state in natural-resource issues (Franco, Gaspard, and Mueller 2019; Gaspard and Missemer 2022).

Another, hitherto undisclosed illustration of the rich relationship between Hotelling's theoretical economics and empirical realities is given by "Stability in Competition" (1929). The article appears as a particularly stylized exercise, with a geographically linear representation of a market, restrictive assumptions on the inelasticity of demand, and the absence of production costs. The main result of the paper—the tendency to clustering—was generalized by Hotelling to all characteristics of goods, with a tendency to the homogenization of supply in all markets, that is, what has been later called the principle of minimal differentiation or the "Hotelling law."

Right from the 1930s and 1940s, Hotelling's model met with some criticism from the growing literature on competition

[46] The Scope of Mathematics…, op. cit.
[47] Ibid.

[48] Herold to Hotelling, 7 July 1930, HHP, Box 42, "Exploitation of Irreplaceable Assets."

and spatial economics on the basis of possible lack of logical consistency (Palander 1935, Smithies 1941, Ackley 1942), excessive sensitivity to assumptions (Chamberlin 1933, Hicks 1935), and unrealism (Lerner and Singer 1937; Robinson 1941; for a review, see Aydinonat and Köksal 2019). Even if Hotelling did not publicly participate in these further debates, he privately addressed most of the criticisms, arguing, for instance, in a letter to Francis W. Dresch dated May 29, 1937, that, contrary to what some authors, especially Lerner and Singer, "erroneously" thought about his demonstration, competitors could not be considered as "instantaneously movable."[49] In a letter to George J. Stigler dated November 20, 1950, Hotelling confirmed his trust in his initial analysis, writing that he had "no errata for this paper."[50]

At the beginning of his paper, Hotelling gave two reasons for his interest in duopoly theory and spatial competition: the concrete fact that "of all purchasers of a commodity, some buy from one seller, some from another, in spite of moderate differences of price" (1929, p. 41); and the observation that this phenomenon "does not seem to have been generally taken account of in economic theory" (1929, p. 44). In other words, he started by confronting an issue and identifying a gap in the literature to be filled. We can trace this path in the archives. The first draft on the spatiality of industry and commerce dates from November 1924.[51] In addition to preliminary equations, it contains a first sketch of the linear representation of the market that will be found in the 1929 paper, already with a bounded area of length l. Interestingly, the draft is not entitled "monopoly/duopoly" or "spatial competition" but "grocery store problem," which shows how Hotelling conceived his entry point on the matter, that is, as related to a very concrete, daily issue.

The same applies to his idea of clustering. The archives contain a small handwritten memo from the 1920s entitled "Instance of clustering" and providing a few examples such as the "N.Y.C. & Penna R.R." running "trains at nearly the same schedule" and "stores keeping the same hours, instead of staggering them."[52] In later drafts and memos we can find references to Cournot, Moore, and others, showing how Hotelling situated his analysis of concrete observations—the grocery store problem, clustering—in the existing literature.[53]

Many criticisms and extensions of Hotelling's 1929 original model have been based on the relaxation of certain assumptions, such as the inelasticity of demand or on the exploration of further demand functions and market forms and structures (unbounded market, more than two competitors, more than two dimensions, etc.). These developments finally led to the famous invalidation of Hotelling's main result—the tendency to clustering—by Claude D'Aspremont, Jaskold Gabsziewicz, and Jacques-François Thisse (1979), showing that "no equilibrium price solution will exist when both sellers are not far enough from each other" (p. 1145) and that there might be situations in which there is in contrast "a tendency for both sellers to maximize their differentiation" (ibid.), when transportation costs are "quadratic with respect to the distance" (p. 1148). Without going back over all the theoretical developments that followed the 1929 article in the economic literature (see Biscaia and Mota 2013), what is interesting here is to observe that Hotelling himself explored many of

49 Hotelling to Dresch, 29 May 1937, HHP, Box 31, "Location of Competitors."

50 Hotelling to Stigler, 20 November 1950, HHP, Box 5, "Miscellaneous S."

51 HHP, Box 10, "AMS Reports and Correspondence (3)."

52 HHP, Box 31, "Location of Competitors."

53 HHP, Box 26, "Mathematical Economics" and HHP, Box 48, "Mathematical Economics (3)."

these extensions in his own personal unpublished notes from the late 1920s to the late 1950s. In 1936, he drafted a 19-page handwritten pile of notes aiming at generalizing "the conditions of 'Stability in Competition' […] to the case of elastic demand."[54] At some point he even found a mathematical result close to that of D'Aspremont et al.'s, that is, a situation where competitors have the tendency to "repel one another," when "elasticity of demand equal [sic] everywhere to unity." Hotelling, however, crossed out his result, revised his calculations on the following page, and finally concluded, going back to his 1929 outcome, that "the clustering tendency exists."[55]

In addition to demand functions, the other theoretical exploration that Hotelling conducted in his personal notes concerned the forms of markets, beyond the linear shape of the article, which has been a source of strong criticisms. Obviously, in 1929 Hotelling chose a very stylized representation of a market area, both linear and bounded. He was fully aware of the need to investigate other types of markets, as testified by a preliminary draft from 1928 showing a situation of a concentric market for food in the United States, with production affected by "market price minus cost of transport," sometimes even by "market price minus costs of transport and fertilizer," suggesting that he explored configurations with positive production costs.[56] Notes from the 1950s, late in his career, confirm his concern about the matter: a memo dated October 28, 1958 shows a project to "extend the linear model of the 1929 'Stability in Competition' to 2 dimensions, the competitors [being] placed [on] an elliptical market area with constant density of demand."[57] After a series of equations, the memo ends with an amusing "this looks like an error," yet showing Hotelling's lasting interest in the extension of his model to a wider range of market forms.

In this direction, one episode is worth relating as it directly highlights Hotelling's view of the realism of his 1929 model and of the role of mathematics in economics. When he submitted his paper in 1928, he corresponded with John Maynard Keynes, who was at the time the editor of the *Economic Journal*. Keynes wrote in a letter dated August 7, 1928, that he found the first version of the paper "decidedly interesting" and almost ready for publication.[58] He only made three remarks: he asked for (i) the addition of a discussion of Piero Sraffa's 1926 article on competition, which Hotelling apparently was not aware of; (ii) a qualification about monopoly profits in section 2; and (iii) he wondered about the validity of Hotelling's point with more than two producers, especially along a circular "chain" where "a neo-coming will find it to his advantage to take up a position midway between two existing firms."[59] Hotelling took the first two remarks into account, changing his paper accordingly. Regarding the circular chain, the archives contain a draft dated September 1, 1928 and entitled "Competitors in a circle," in which Hotelling directly tackled Keynes's point.[60] Supposing "*n* competitors

[54] Location of Competitors, November 21, 1936, HHP, Box 31, "Location of Competitors." This document seems to have been written by Hotelling in reaction to Lerner and Singer's still unpublished 1937 paper. We know from the archives (Frisch to Hotelling, 27 June 1935, HHP, Box 1, "Frisch, Ragnar") that Hotelling had been asked to review the paper for *Econometrica*, suggesting major revision. Lerner and Singer preferred to submit the paper to the *Journal of Political Economy* without taking the remarks into account, which rather displeased Hotelling (Hotelling to Viner, 27 May 1936, HHP, Box 31, "Location of Competitors").

[55] Ibid.

[56] HHP, Box 26, "Mathematical Economics."

[57] HHP, Box 45, "Misc. Problems, Dated 1957–1959."

[58] Keynes to Hotelling, 7 August 1928, HHP, Box 42, "Calculations Relating to: 'Differential…'."

[59] Ibid. The last two remarks are handwritten on the back of the letter.

[60] HHP, Box 42, "Calculations Relating to: 'Differential…'." It is quite fascinating to see that Keynes

[…] placed on a circular line along which there is a continuous and uniform distribution of an inelastic demand," he concluded that, in terms of location choice, "as Keynes says, the drag to the left equals the drag to the right."[61] In his response letter, Hotelling further added that his result might be different "after taking account of the elasticity of demand," this situation falling outside the scope of his paper.[62]

The most interesting insight coming from this episode is the reason why Hotelling decided not to include the circular chain, even with inelastic demand, in his final article. In his response, he argued: "though presenting some possible *mathematical* interest, [a circular chain of competitors] does not appear to be sufficiently important for *economics* to warrant publication in the *Economic Journal*" (our italics).[63] In other words, the circular chain might be interesting from a mathematical viewpoint but was not very relevant for economists. Why? Hotelling made his point clear: market areas with "no boundary" and "homogeneous" commodities "must in practice be extremely rare."[64] This observation is particularly important because it shows that, in spite of the trials of unrealism suffered by his model, Hotelling considered that it represented likely situations. Mathematics for its own sake had no interest for him if it did not correspond to concrete economic situations.

Other scattered notes in Hotelling's archives support this interpretation of a 1929 contribution preserving some form of realism. Part of the criticism of the model has been related to the lack of strategic anticipation on the part of competitors. In the subsequent history of economics, game theory enriched the analysis of duopoly and imperfect competition. D'Aspremont, Gabszewicz, and Thisse's 1979 framing of their argument as "a two-person game with *players* […], *strategies* […], and *payoff functions*" (p. 1145, italics in the original) is a prime example. Hotelling's archives contain a memo, dated October 1946, on John von Neumann and Oskar Morgenstern's *Theory of Games and Economic Behavior* (1944), that reads: "[…] applied to the duopoly situation of 'Stability in Competition' (HH 1929) […], the solution of von Neumann & Morgenstern appears highly unrealistic for this case."[65] Again, Hotelling left out possible reinterpretations of his problem in the name of realism: it was unlikely, according to him, that duopolists would behave as fully rational and strategic players; there was therefore no need to push the investigation too far in this direction.

At the end of the day, it appears that Hotelling himself explored many developments of his 1929 model alongside the growth of the literature on competition theory and spatial economics, from the 1930s to the 1960s. He had circular markets in mind 25 years before Chamberlin (1953) and situations where competitors might repel one another almost 50 years before D'Aspremont, Gabszewicz, and Thisse (1979), although he did not complete all his extensions. As recently as 1966, he kept considering that his model could be extended to more than two dimensions, with more than two competitors, with robust results.[66] This does not mean that he single-handedly covered all the subsequent developments in the field, or that he was always right, far from it—the model did contain logical flaws and

and Hotelling explored circular markets almost 25 years before Chamberlin (1953) used them as an allegedly new kind of criticism addressed to Hotelling (1929).

[61] Ibid.

[62] Hotelling to Keynes, 7 September 1928, HHP, Box 42, "Calculations Relating to: 'Differential…'."

[63] Ibid.

[64] Ibid.

[65] HHP, Box 48, "Mathematical Economics (2)."

[66] Hotelling to Etz, December 30, 1966, HHP, Box 3, "Miscellaneous E."

today's literature is obviously richer. But it shows that, in his mind, his initial model was to be completed, and most interestingly, that it was much more realistic than has often been said, opting for a representation of markets and of competitors' behaviors that is more plausible than many alternatives subsequently given in the literature. What mattered was building a model, not for the sake of theoretical, abstract economics, but to solve the grocery store problem and improve our understanding of the concrete homogenization of supply in markets. For Hotelling, the 1929 model, albeit upgradeable, was already deeply helpful on this matter.

The two examples given above—Hotelling (1929) and Hotelling (1931b)—are quite representative of Hotelling's general conception of the articulation between theory and empirics in economics. Obviously, most of the hypotheses of his basic formalizations were restrictive but he often explored, in his unpublished drafts, extensions and developments, sometimes to conclude, such as in the 1929 case, that the absence of some complexities (e.g., strategic behavior, expectations) were more in line with reality than what economists often think. It is also interesting that today's literature tends to reduce Hotelling's achievements to a few principles, such as the Hotelling rule for his 1931 paper. A careful reading of his articles and of his accompanying unpublished notes show that his work went far beyond this, with constant attention to the deviations of reality from his basic results. This is why he did not consider his 1931 rule as applicable to concrete energy markets, but only to pure irreplaceable assets. In other words, the Hotelling rule, the Hotelling law, the Hotelling lemma and so on should have not been taken as Hotelling's *results*, rather as the *starting-points* of his analyzes. Moreover, these principles could not be separated from their finalities, namely the clarification of concrete economic situations. Hotelling is often remembered for his mathematical economics; we should not forget that, in his mind, mathematical economics was above all an applied science.

4.2 *Georgism, Market Socialism, and Economic Reforms*

Throughout his career, especially in his academic writings, Hotelling never made his political orientation explicit. As shown above, his papers were designed to compare different social organizations (liberalism and socialism) or economic policies that had analogous statuses, some configurations having more merit than others depending on the objectives targeted by society. Hotelling was deeply involved in academic life, in his successive universities, and in scholarly societies (e.g., the Econometric Society). In contrast, except during the Second World War, he rarely participated in committees or in the preparation of reports commissioned by public authorities. And he never officially joined a political party. In brief, it appears that direct involvement in politics was not part of his credo.

The analysis of his overall career and of his research priorities, however, questions the role played by one particular intellectual and political current of the turn of the twentieth century: Georgism. A large number of clues suggest that Hotelling was quite close to Georgist ideas and that, without discrediting the scientific rigor of his work, these penchants may have contributed to the structuring of his research agenda. Hotelling's contributions should thus also be read in the light of this background.

Henry George was a nineteenth-century economist and politician whose ideas became enormously popular following the publication of his book *Progress and Poverty* (1881). He founded a movement, Georgism, characterized by progressive ideas and imbued with Methodist and reformist values. Basically, George considered that a large part of economic wealth should be credited to the work of nature, that is, a divine gift belonging to everyone indiscriminately. He was a fervent opponent of rent, especially on land, considering also that rent was often the result of speculation. His idea was to tax (even confiscate) rent, in order to redistribute wealth and eliminate the endemic problem of extreme poverty that plagued a significant segment of the American population. The overall fiscal system was intended to rest upon this single rent tax, with a return large enough to cover all public expenses. Such a redistribution was seen by George as both morally just and economically efficient.

In George's view, the rent from agricultural land was the most important subject to be dealt with. The argument, however, was easily extendable to rents derived from natural monopolies such as electricity, railways, and natural resource exploitation. Inheritance, in a sense, could also be considered as a form of rent—not based on individual merit—to be socialized. Georgism did not oppose the free market (Barker 1955). Any wealth resulting from the latter was seen as legitimate and efficient as long as it came from merit. The fruits of labor remained, with a single tax (on rent), entirely in the hands of those who worked, competition in free markets ensuring that what was earned was equivalent to the effort provided. The redistribution mechanism concerned only those gains that—from a Georgist point of view—came from an illegitimate appropriation of nature's work. The Georgist argument was thus certainly aimed at preventing extreme poverty but in no way opposed even colossal gains, as long as they came from labor and not from rent.

In the 1910s, Georgist ideas contributed to Hotelling's education, right from his arrival in Seattle. George F. Cotterill, Seattle's then mayor, proposed a single-tax reform to his citizens in 1912. It was rejected but the debates in the city between Georgists and opponents continued for some time (England 2015, 2016).

It is difficult to find, either in the published material or in the archives, an explicit claim by Hotelling of Georgist ideas. His participation in the foundation of the *American Journal of Economics and Sociology*, which claimed Georgist affiliation, is the most straightforward indication we can find (Dewey 1941, Lissner 1974). The archives reveal that Hotelling sometimes took part in local wrangles against land speculation, yet without directly referring to George.[67]

Nevertheless, the way he framed many of his research questions retrospectively appears to be reminiscent of a Georgist agenda. Despite impressive popular success, George was despised among American economic scholars (Furner 1979, Ross 1991, Henry 1995, England 2023). In a sense, Hotelling implicitly attempted to rescue George's reputation as an economist by providing sound theoretical credentials to many Georgist ideas (Gaffney 1972, Mueller 2021).

His major papers can indeed be read as welfare comparisons between Georgism and capitalism, often showing the superiority of the former system. Hotelling adapted the concept of consumer surplus to situations of general (possibly intertemporal) equilibrium, interpreting these generalizations as a monetary measure of well-being. In 1929, he compared free trade and a planned economy, concluding that the latter was more efficient both in allocating resources and in providing a diversity of products—"an argument to the socialist side" (1929, p. 52). In 1931, he compared private and public ownership of exhaustible resources, and presented different taxation policies as an option to slow down

[67] HHP, Box 51, "Clippings" Regarding land speculation, Darnell (1988, 59) relates that Hotelling liked playing the game of Monopoly, by adjusting the rules. This is probably not just an anecdote, since Monopoly seems to have influenced the way Hotelling framed some of his research questions, and since the inventor of the first version of the game, Elizabeth Magie, was a feminist and Georgist activist (Orbanes 2006, Pilon 2015, see Mueller 2021).

exhaustion, particularly praising severance taxes as a suitable instrument that may both slow exhaustion and decrease prices, "to be commended if the monopolist is regarded as *unfairly* possessed of his property" (1931b, p. 167, italics in the original). Such a tax would confiscate part of the monopolist's rent, thus acting analogously to a Georgist single tax. Hotelling's paper on the Edgeworth paradox (1932) can also be read as an attempt to wisely tax a monopolist in a way that would reduce his rent, increase the public budget, and reduce prices, thus increasing general welfare. Again, such a tax would mimic, in a sense, some features of a single tax. Finally, in 1938, endorsing a classical Georgist argument, Hotelling declared himself in favor of the superiority of income, inheritance, and land taxes over excise taxes to get "a state more satisfactory […] than before" (1938, p. 252).

Beyond Georgism, the 1938 article is particularly insightful in exploring the connection between Hotelling's economics and political reforms. Hotelling became directly involved in debates about the implementation of marginal cost pricing. As mentioned above, the archives reveal that he endorsed marginal cost pricing from the early 1930s,[68] when he directly linked its necessity to imperfect competition. Two years later, he presented his first results in Colorado Springs under the title "A Basic Defect of Capitalism."[69] At the turn of the 1950s, he exchanged ideas and proposals on the matter with French economists Maurice Allais, Marcel Boiteux, and, to a lesser extent, Gérard Debreu.[70]

Allais contacted Hotelling at the end of the Second World War suggesting the presence of an inconsistency in his 1938 article, which turned out instead to be a trivial miscalculation on the Frenchman's part. However, the exchange took on broader dimensions, focusing in particular on the calculation of the deadweight loss, with the increasingly conspicuous participation of Debreu, who would eventually take his own independent position and introduce new topological tools into the analysis of the deadweight loss (Fèvre and Mueller 2023).

It is interesting that the discussion between Hotelling and his French colleagues finally had practical implications since, on the basis of Allais's arguments, Boiteux developed marginal pricing of electricity (Yon 2016, 2020). We know from travel notes available in the archives that Hotelling visited Allais and Boiteux in Paris in 1951 to continue the discussion.[71] He then constantly kept himself informed of the work of Boiteux, Pierre Stasi, and Pierre Massé, which suggests he was interested in the concrete implementation of marginal cost pricing. The archives contain reprints of texts published at the time (e.g., Boiteux 1950, Boiteux and Stasi 1952) and related personal notes wondering, for instance, how "fixed costs [can] be covered" taking into account the fact that "making different rates according to time of day is considered impractical."[72]

In the end, Hotelling did not limit himself to building a platform to compare different policy options, thanks to the support of advanced mathematics. When he had occasions to demonstrate the merits of Georgist-inspired policies, he took them all. This is not to say that his results were biased—it is quite clear that he did not want his background preferences to interfere with the results of his models. Nonetheless, the way he framed his research questions about

68 Prosperity Through Increased Production…, op. cit.
69 HHP, Box 25, "The General Welfare to 1938."
70 HHP, Box 56, "Printed Material."
71 HHP, Box 22, "UNC European Trips 1951, 1953."
72 HHP, Box 46, "Lectures (2)." As mentioned, Hotelling also indirectly participated in the marginal cost controversy in the United States (Frischmann and Hogendorn 2015, Desmarais-Tremblay et al. 2023).

capitalism versus socialism, free competition versus monopoly, and general welfare versus private profits should also be read through the lens of Georgist ideas that at least influenced the choice of topics he worked on. Despite their high technicality and apparent universality, Hotelling's contributions to economic analysis also need to be understood in their historical, political, and moral context.

5. *Conclusion*

While Harold Hotelling's most visible results in economics cover apparently unrelated topics such as spatial economics, exhaustible resources, and welfare economics, our inquiry shows that they all have their roots in the mid-1920s, as several branches of a common project: to explore how the use of an enlarged spectrum of mathematical tools could endow economics with transparent reasoning, general results, and sound indicators for welfare valuations. The approach was, however, anything but abstract for Hotelling, rather being driven by the desire to enlighten public debate in years that saw major transformations in the American economy, questioning the legitimacy of laissez-faire, the need (or not) to regulate business affairs (from grocery stores to big railroad or mining companies), the best ways to organize public utilities and to tax activities. Each branch took several years to flourish, Hotelling seeking to establish clear-cut mathematical results without sacrificing the empirical relevance of his analysis. He wrote hundreds of drafts, collecting data and observations, searching for appropriate ways to put them into algebraic forms to be manipulated with the calculus of variations and differential geometry, eliminating that which appeared meaningless regarding daily preoccupations.

Hotelling's economics can be defined as applied mathematical science for policy-making, with a personal obsession for the "general welfare," or put differently, for "what is good [...] for mankind in general, as opposed to the problems which most people find more immediately interesting regarding the means of achieving their own individual aims."[73] Because of his education, his background, and the context in which he wrote, Hotelling designed a research agenda not only on the basis of the heuristic interest of theoretical questions, but also in relation to preoccupations aligned with Georgism. His commitment to the community was a permanent feature of his career, from his early interest in journalism, when he thought it could help him "stimulate proper action on public matters," to his "study of economics [to make] some changes in the institutions."[74]

Hotelling contributed to American economics as much as a teacher and promoter of mathematics in the social sciences as he did as a researcher. As mentioned, his archival materials show that he taught both classical mathematical economics and up-to-date research, including his own work.[75] His Georgist inclinations were known by his students. Many started their careers by further exploring Hotelling's research themes (e.g., Preinreich 1938, 1940; Pabst 1940, 1942; Vickrey 1944, 1945). Others followed his search for mathematical results as proofs of possibility or impossibility (e.g., Arrow 1950, 1951) and, more profoundly, his tendency to identify "welfare economics as pointing to an ideal efficient economy [and] socialism [as] the way in which the ideal market was to be achieved" (Arrow 1978, p. 476). Even if other factors and inspirations certainly came into play, it should not come as a surprise, in a sense, if some of Hotelling's students paid

[73] Address to the Econometric Society, December 28, 1937, HHP, Box 25, "The General Welfare to 1938," 1.

[74] Western Hotelling…, op.cit., 17.

[75] HHP, Box 48, "Mathematical Economics (1)." Some of Hotelling's syllabi are also available at http://www.irwincollier.com/columbia-economics-course-descriptions-hotelling-1931-1945/

so much attention to Georgism throughout their careers—the example of Vickrey is significant in this regard (Wenzer 1999).

Not only did Hotelling teach many promising students at Columbia University, he also supported young scholars from other institutions, such as Debreu, as shown above, and Samuelson,[76] who would later identify his predecessor as one of the "heroes" of the economics discipline (1972, p. 253). Hotelling therefore played an active role in the emergence of mathematical economics after the Second World War, in the United States and beyond, by training and encouraging young scholars with bright futures in the profession.

In the 1920s, 30s, and 40s, the discipline of economics was still searching for its best heuristic pathway, between institutionalism, the rise of statistics and econometrics, and renewed mathematical economics. In those unsettled times, Hotelling did not deviate from his route, paving the way for an idiosyncratic approach to economic analysis, mixing advanced mathematics with policy relevance and special attention to empirical realities. Today, at a time when economics is again seeking to reinvent itself, both theoretically, in relation to the major challenges of our time (e.g., climate change, geopolitical instability, rising inequalities, deglobalization), and methodologically (e.g., big data, artificial intelligence, machine learning), Hotelling's promise to combine the most advanced research tools with the societal relevance of the proposals remains an unparalleled source of inspiration.

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[76] Hotelling suggested to *Econometrica* that they make room for Samuelson's early contributions (Hotelling to Leavens, 9 February 1938, HHP, Box 13, "Econometrica").

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